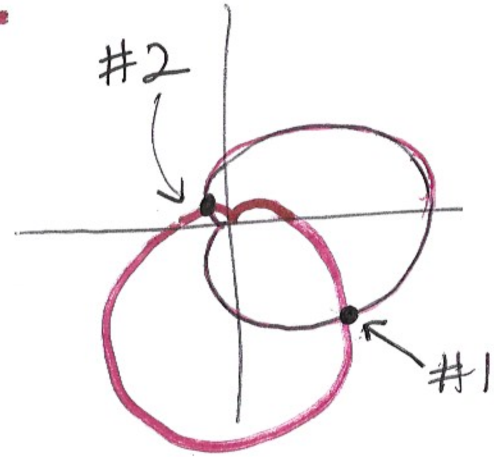
$r = 1 + \cos\theta$

$r = -1 - \sin\theta$

$\theta \in [0, 2\pi]$



$r = 0$?

$1 + \cos\theta = 0$ $\quad$ $-1 - \sin\theta =$

$\cos\theta = -1$ $\quad$ $\sin\theta =$

$\theta = \pi$ $\quad$ $\theta = 3$

INTERSECTION @ P

$$1 + \cos\theta = -1 - \sin\theta$$

$$(\cos\theta + \sin\theta)^2 = (-2)^2$$

$$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 4$$

$$1 + \sin 2\theta = 4$$

$$\sin 2\theta = 3$$ IMPOSSIBLE (NO SOL'N)

OR $(2 + \cos\theta)^2 = (-\sin\theta)^2$

$$4 + 4\cos\theta + \cos^2\theta = \sin^2\theta = 1 - \cos^2\theta$$

$$2\cos^2\theta + 4\cos\theta + 3 = 0$$

$$\cos\theta = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-4 \pm \sqrt{-8}}{2} \notin \mathbb{R} \longrightarrow \text{NO } \theta$$

(r, θ) IS THE SAME POIN

$(r, \theta + 2n\pi)$ AND $(-r, \theta + \pi$

$\downarrow$

(r, θ) IS THE SAME POINT A

$(-r, \pi + \theta)$

NEED TO SUB $(-r, \pi + \theta)$

INTO EQ'NS 1 AT A TIME

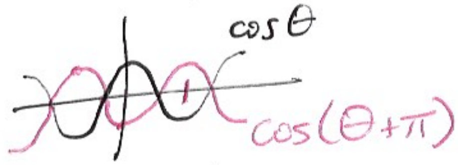
$(-r, \pi+\theta)$ IN $r=f(\theta)$
(r, θ) IN $r=g(\theta)$ $\theta\in[0,\pi]$

(r, θ) IN $r=f(\theta)$
$(-r, \pi+\theta)$ IN $r=g(\theta)$ $\theta\in[0,\pi$

$$r = 1+\cos\theta \xrightarrow{(-r,\pi+\theta)} -r = 1+\cos(\pi+\theta) \longrightarrow -r = 1-\cos\theta \longrightarrow r = -1+c$$

$$r = -1-\sin\theta$$

COS θ SHIFTED LEFT BY π



$$-1+\cos\theta = -1-\sin\theta$$
$$\cos\theta = -\sin\theta$$
$$-1 = \tan\theta \longrightarrow \theta = \frac{3\pi}{4}$$

CHECK: $r = 1+\cos\left(\frac{3\pi}{4}+\pi\right)$

$$r = 1+\cos\frac{7\pi}{4} = 1+\frac{\sqrt{2}}{2}$$
$$r = -1-\sin\left(\frac{3\pi}{4}\right)$$
$$r = -1-\frac{\sqrt{2}}{2}$$

$\left(1+\frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right)$ ON $r = 1+\cos\theta$

$\left(-1-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ ON $r = -1-\sin\theta$

SAME POINT

IE, INTERSECTION POINT #1

$r = 1+\cos\theta$ $\qquad\qquad\qquad \theta \in [0,\pi]$

$r = -1-\sin\theta \xrightarrow{(-r,\pi+\theta)} -r = -1-\sin(\pi+\theta) \longrightarrow -r = -1-{}^{-}\sin\theta \longrightarrow r = 1-\sin$

$1+\cos\theta = 1-\sin\theta$ $\qquad$ CHECK: $r = 1+\cos\frac{3\pi}{4}$

$\cos\theta = -\sin\theta$ $\qquad\qquad$ $r = 1-\frac{\sqrt{2}}{2}$

$-1 = \tan\theta \quad \theta = \frac{3\pi}{4}$

$r = -1-\sin\left(\frac{3\pi}{4}+\pi\right)$

$r = -1-{}^{-}\frac{\sqrt{2}}{2} = -1+\frac{\sqrt{2}}{2}$

SAME POINT
IE. INTERSECTION
POINT #2

$\left(1-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ ON $r = 1+\cos\theta$

$\left(-1+\frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right)$ ON $r = -1-\sin\theta$

TO FIND INTERSECTIONS OF $r=f(\theta)$ WITH $r=g(\theta)$

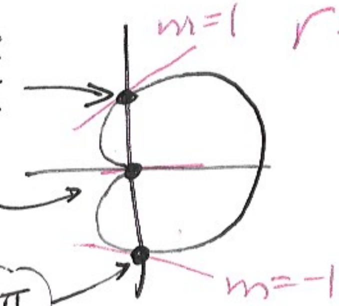
1. SOLVE $f(\theta)=0$ AND $g(\theta)=0$ FOR $\theta\in[0,2\pi]$
   MAY HAVE DIFFERENT SOL'NS
2. SOLVE $f(\theta)=g(\theta)$ FOR $\theta\in[0,2\pi]$
3. REPLACE (r,θ) WITH $(-r,\pi+\theta)$ IN $r=f(\theta) \longrightarrow r=F(\theta)$
   AND INTERSECT WITH $r=g(\theta)$ IE. SOLVE $F(\theta)=g(\theta)$ FOR θ
4. REPLACE (r,θ) WITH $(-r,\pi+\theta)$ IN $r=g(\theta) \longrightarrow r=G(\theta)$
   SOLVE $f(\theta)=G(\theta)$ FOR $\theta\in[0,\pi]$

FIND SLOPE OF TANGENT LINE AT y-INT OF $r = 1 + \cos\theta$

$$\underbrace{\text{FIND SLOPE OF TANGENT LINE}}_{\frac{dy}{dx}}$$

$r = f(\theta)$

$\theta = \frac{\pi}{2}$



$r = 0 = 1 + \cos\theta$ ← $\theta = ?$

$\cos\theta = -1$

$\theta = \pi$

$\theta = \frac{3\pi}{2}$

$$\left.\begin{aligned} x &= r\cos\theta = f(\theta)\cos\theta \\ y &= r\sin\theta = f(\theta)\sin\theta \end{aligned}\right\}$$ PARAMETRIC EQUATIONS IN PARAMETER θ

PRODUCT, N

$\frac{d}{d\theta}\sin\theta$

$$\frac{dy}{dx} = \underbrace{\frac{\frac{d}{d\theta} f(\theta)\sin\theta}{\frac{d}{d\theta} f(\theta)\cos\theta}}_{\text{FROM } \frac{dy}{dx} \text{ IN PARAMETRIC}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$x = (1 + \cos\theta)\cos\theta$

$y = (1 + \cos\theta)\sin\theta$

$$x = (1+\cos\theta)\cos\theta$$

$$y = (1+\cos\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin^2\theta + (1+\cos\theta)\cos\theta}{-\sin\theta\cos\theta - (1+\cos\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-1^2 + \cancel{(1+0)0}}{\cancel{-1(0)}-(1+0)\cdot 1} = 1$$

$\cos\frac{\pi}{2} = 0$

$\sin\frac{\pi}{2} = 1$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = \frac{-(-1)^2 + \cancel{(1+0)0}}{\cancel{-(-1)0}-(1+0)(-1)} = -1$$

$\cos\frac{3\pi}{2} = 0$

$\sin\frac{3\pi}{2} = -1$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\cancel{-0^2} + \cancel{(1+-1)(-1)}}{\cancel{-0(-1)}-\cancel{(1+-1)0}} = \frac{0}{0}$$

DOESN'T NECESSARILY MEAN $\frac{dy}{dx}$ DNE

$\cos\pi = -1$

$\sin\pi = 0$

$$\lim_{\theta\to\pi}\frac{dy}{dx} = \lim_{\theta\to\pi}\frac{-\sin^2\theta + \cos\theta + \cos^2\theta}{-\sin\theta\cos\theta - \sin\theta - \sin\theta\cos\theta}$$

$$= \lim_{\theta\to\pi}\frac{\cos 2\theta + \cos\theta}{-\sin 2\theta - \sin\theta} \qquad \frac{0}{0}$$

L'H

$$= \lim_{\theta\to\pi}\frac{-2\sin 2\theta - \sin\theta}{-2\cos 2\theta - \cos\theta} \qquad \frac{0}{-2(1)-(-1)} = \frac{0}{-1} = 0$$

$$= 0$$